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DEPENDENCE OF THE EFFICIENCY OF A CO_2 GASDYNAMIC LASER (GDL) RESONATOR ON THE LASING MIXTURE PARAMETERS

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Carbon dioxide gasdynamic lasers are widely studied at this time [1]. Numerous methods for analyzing the GDL characteristics have been developed, starting with approximate analytical formulas permitting execution of some estimates, to numerical methods of solving complex systems of differential equations describing diverse physical processes. Nevertheless, perfecting the analytical formulas remains urgent. This is related to the ongoing search to raise GDL efficiency by application and development of new methods to obtain an active medium, which is related, in turn, to the need to optimize many parameters.

An analytic dependence is obtained in this paper for the limit value of the resonator efficiency (understood here to be the ratio between the number of radiation quanta which have emerged from the resonator and the number of vibrational quanta accumulated at the upper lasing level and in the nitrogen molecules) as a function of the characteristics of the active medium at the resonator input with relaxation losses taken into account in its cavity and without thermodynamic equilibrium between the vibrational modes (a four-temperature model). The conditions of equality of the total radiation losses and the total amplification for a constant intensity in the whole resonator volume [2]

$$2\langle k\rangle d = \ln(1/r), \tag{1}$$

is used in the derivation, where $\langle k\rangle = (1/S)\int k\,dS$; k is the gain coefficient, S is the area of the generation zone, r is the effective coefficient of resonator reflection taking into account the losses associated with absorption in the mirrors and the radiation yield, and d is the thickness of the active medium along the optical axis. An explicit expression for the power being generated is obtained in [2] within the framework of the two-temperature model. Analysis of this expression and optimization of certain resonator parameters permit limit values to be obtained for the parameter efficiency σ for a given ratio η between the gain of the active medium k_0d at the resonator input and the absorption coefficient δ of the mirrors:

$$\sigma = 1 - (1 + \ln\eta)/\eta. \tag{2}$$

However, the formula presented in [2] does not correctly indicate the nature of the dependence of the power of the radiation being generated on the composition of the lasing mixture (for instance, its maximal value is obtained in the absence of water vapor), and on the length of the generation zone, for which the power grows continuously, according to [2], as it diminishes. This is a result of using the two-temperature model which is not applicable for large values of the radiation intensity in the resonator and a small relaxation rate of the lower lasing level, which takes place for a deficiency of vapors. All this results in the need to consider the four-temperature model. Linearized equations describing a system in conformity with this model are represented in [2], and the means to solve the problem are noted. In this paper the solution is executed to an explicit expression for σ and it is analyzed. A system of linear equations analogous to [2] is used

$$de/d\xi = A\mathbf{e} + \mathbf{B}, \tag{3}$$

$$\mathbf{e} = (e_1, e_2, e_3), \quad \mathbf{B} = (K_1 e_{1*}, 0, 0),$$

$$A = \begin{pmatrix} -\left(K_1 + \beta_1 I + 3\beta_2^2 K_{1,2}\right) & \beta_1 I + 3\beta_2 K_{1,2} & 0 \\ \beta I + K_{1,2}\beta_2 & -(x_N + \beta I + K_{1,2}) & x_N \\ 0 & x_C & -(x_C + x_H K_3) \end{pmatrix},$$

where $\xi = x/v\tau_{ex}$; $\beta = K_0\tau_{ex}[x_C N(e_2(0) - e_1(0))h\nu]^{-1}$; $K_i = \tau_{ex}/\tau_i$, $\beta_1 = 2\beta\sqrt{e_{1*}}$; $\beta_2 = \sqrt{e_{1*}}$. Here e_3, e_2, e_1 are the populations of the vibrational modes of nitrogen, the upper and lower lasing levels in the computation per molecule of the appropriate species, x_C, x_H, x_N are the carbon dioxide gas, water vapor, and nitrogen con-

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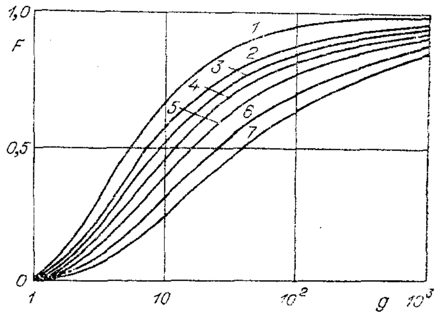

Fig. 1

centrations in the mixture, I is the radiation intensity, e_{1*} is the equilibrium value of e_1, τ_{ex} is the time of vibrational quanta exchange between the N_2 and CO_2, and τ_1, $\tau_{1,2}$, τ_3 are the vibrational relaxation times of the corresponding modes. The equations (3) differ from the equations in [2] by the presence of the term describing the V−T relaxation of the nitrogen vibrations upon collision with the H_2O molecules, as well as a modified term corresponding to the V−V exchange between the upper and lower lasing levels.

It can be shown that for a CO_2 laser with moderate carbon dioxide gas concentration ($x_C < 0.2$), one of the eigennumbers of the matrix A is much less in absolute value than the other two, and all three are negative. Consequently, the contribution of the eigenvectors of the matrix A corresponding to large eigennumbers in absolute value, can be neglected in considering the resonator integral characteristics, and it is possible to limit oneself to investigation of the slowly decreasing solution. It can also be shown that the minimal eigennumber λ is much less in absolute value than the diagonal elements of the second and third rows of the matrix A. This simplifies finding λ and its corresponding eigenvector (c_1, c_2, c_3). Thus, substituting the solution

$$\mathbf{e} = \mathbf{c}\exp(-\lambda\xi) - A^{-1}\mathbf{B}$$

in the first and second equations of the system (3), and neglecting the quantity λ in comparison with the diagonal elements, we obtain the values of the eigenvector components

$$\mathbf{c} = \mathrm{const}\begin{pmatrix} x_N(\beta_1 I + 3\beta_2 K_{1,2}) \\ x_N(K_1 + \beta_1 I + 3\beta_2^2 K_{1,2}) \\ \beta I[K_1 + 2\beta_2 x_N - \beta_2(1-\beta_2)K_{1,2}] + K_1 K_{1,2} + x_N(K_1 + 3\beta_2^2 K_{1,2}) \end{pmatrix}.$$

Knowing $\mathbf{c}$, we can find from the first equation

$$\lambda = x_C \frac{\beta I[K_1 - \beta_2(1-\beta_2)K_{1,2}] + K_1 K_{1,2}}{\beta I[K_1 + 2\beta_2 x_N - \beta_2(1-\beta_2)K_{1,2}] + K_1 K_{1,2} + x_N(K_1 + 3\beta_2^2 K_{1,2})} + x_H K_3.$$

The magnitude of the intensity I is found from the condition (1)

$$\frac{c_2 - c_1}{c_3}\frac{1}{\lambda L}[1 - \exp(-\lambda L)] + K_1 \frac{e_{1*}}{e_3(0)}(\mathrm{Det}\,A)^{-1}[x_H K_3(x_N + K_{1,2} - \beta_2 K_{1,2}) + x_C K_{1,2}(1-\beta_2)] = \frac{e_2(0) - e_1(0)}{2dk_0 e_3(0)}\ln(1/r).$$

Correspondingly, the power W being generated equals [1]:

$$W = (t/2)IS,$$

where t is the transmission factor. Maximizing the quantity W with respect to t and L, we find the limit value of the resonator efficiency for given working gas parameters at the input:

$$\begin{gathered} \sigma = fF(g, \gamma), \\ f = x_C a_1 (x_C a_2 + x_H K_3 a_3)^{-1}, \quad g = a_1 a_4^{-1}\eta, \\ \gamma = (x_H K_3 a_4 + x_C x_N^{-1} K_{1,2}) a_3 a_4^{-1} (x_H K_3 a_3 + x_C a_2)^{-1}, \\ a_1 = 1 - 3\nu_1, \quad a_2 = 1 - \nu_1, \quad \nu_1 = \beta_2(1-\beta_2)K_{1,2}K_1^{-1}, \\ a_3 = 1 + 2\beta_2 x_N K_1^{-1} - \nu_1, \quad a_4 = 1 + K_{1,2}(x_N^{-1} + 3\beta_2^2 K_1^{-1}), \end{gathered} \tag{4}$$

where the function $F(g, \gamma)$ is expressed in parametric form

$$\begin{cases} F = [1 + \gamma\varphi/(g-\varphi)]^{-1}[1 - (1 + \ln\varphi)/\varphi] \\ \gamma = \dfrac{(g-\varphi)^2}{g\varphi}\,\dfrac{\ln\varphi}{\varphi - 1 - (2 - \varphi/g)\ln\varphi} \end{cases} \quad \varphi \leqslant g.$$

The expression (4) has been obtained under the assumption $e_3(0) \gg e_{1*}$.

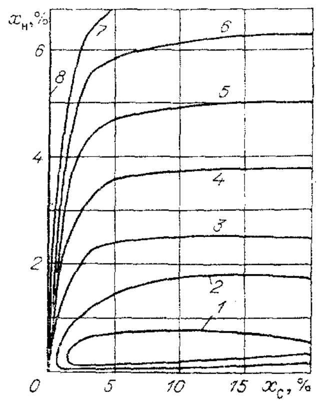

Fig. 2

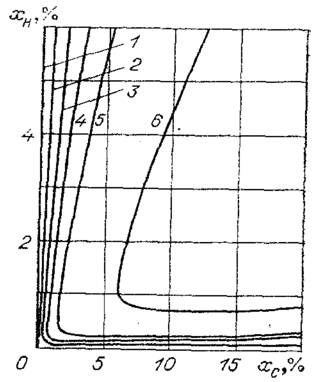

Fig. 3

A graph of the function $F(g, \gamma)$ ($\gamma=0$; 0.02; 0.04; 0.1; 0.2; 0.5; 1 are curves 1-7, respectively) is displayed in Fig. 1. It is seen from the graph that for given g the value of F will be the greater, the smaller the γ. For $f=1$, $\gamma=0$ the expression (4) goes over into (2). This is related to the fact that when there is no vibrational relaxation of the upper lasing level and nitrogen ($K_{1,2}=0$, $K_3=0$), the resonator efficiency will grow as L grows, hence I will diminish and the four-temperature model will become equivalent to the two-temperature model.

The quantity γ is a function of the temperature and the mixture component concentration. Let us note that as x_C changes, the value of η also changes. The dependence of γ on x_C and x_H is shown in Fig. 2 for the temperature 350°K ($\gamma=0.04$; 0.05; 0.075; 0.1; 0.125; 0.15; 0.175; 1 are curves 1-8, respectively).

The data in [3-5] are used to evaluate the constants K_1, $K_{1,2}$, K_3 at this temperature:

$$K_3 = 1.4 \cdot 10^{-2},$$
$$K_1 = 2.0 \cdot 10^{-2} x_C + 1.2 \cdot 10^{-2} x_N + 24 x_H,$$
$$K_{1,2} = 2.6 \cdot 10^{-2} x_C + 0.7 \cdot 10^{-2} x_N + 2.0 x_H.$$

The dependence of the function f on the working gas composition is represented in Fig. 3 ($f=0$, 0.9, 0.95, 0.97, 0.98, 0.99 are curves 1-6, respectively). It is seen from the graphs that for fixed values of the translational and vibrational temperatures at the resonator input, as well as for the ratio d/δ as x_C grows, and therefore, as η grows for an optimal selection of x_H, the resonator efficiency increases. However, this growth occurs more slowly and a regime near saturation sets in for higher values of x_C than follows from an examination of the two-temperature model without taking account of upper level relaxation. The dependence of the efficiency of systems producing the active medium on the gas composition and temperature is different in nature: As a rule the efficiency decreases as x_C increases, consequently, optimization of the GDL parameters as a whole should be performed with their influence on the efficiency of all the generator elements. Formula (4) can be applied in the formulation of such problems.

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